Numerical Study of Cr^{4+}:YAG Passively Q-switching Nd:$GdVO_4$ Laser

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Theoretical analysis is presented to optimize the performance of passively Q-switched Nd:$GdVO_4$ laser with Cr^{4+}:YAG as a solid state saturable absorber. A simple model is developed based on numerically solving the passive Q-switching rate equations, where the impact of the saturable absorber excited states photon absorption is included. The effects of both the saturable absorber doping concentration and output coupler reflectivity on the output laser characteristics are studied. The simulated results show reasonable agreement with those obtained experimentally by other research groups.

1. Introduction:

High energy laser pulses of short durations may be obtained by Q-switching techniques, where energy is stored in the gain medium through optical pumping while the quality factor of the laser resonator is decreased to prevent laser oscillation. Compared with active Q-switching, passive Q-switching is more economical and practical because of the modest requirement of optical elements inside the laser cavity. Consequently, these techniques have attracted much research regarding their design and theory [1-19]. The use of solid state saturable absorber (SA) as a passive Q-switching of diode pumped solid state lasers allows for the development of compact, miniature (microchips), stable and highly efficient laser sources of short bursts and high peak power densities which are desirable for various scientific, medical and military applications. The good thermal, mechanical and optical properties of Cr^{4+}:YAG crystal in addition to its low saturating intensity and high damage threshold allow it to be the most promising Q-switcher SA for many Nd:hosted lasers because of

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its near infrared absorption band (0.9-1.06 μm) [1-4]. Since 1992 $Nd:GdVO_4$ crystal appeared as a new solid state laser gain medium suitable for direct laser diode pumping and excellent for operation as a continuous wave laser and active Q-switching emission [5-9]. Passive Q-switching operation of $Nd:GdVO_4$ laser with Cr^{4+}:YAG SA was demonstrated by Li *et al*. [10] but very low values of pulse energy and peak powers were reported. Very recently, two different research groups [11,12] succeeded in achieved efficient passive Q-switching pulses of larger single pulse energy and higher peak powers using only simple plane-concave resonator. Theoretical treatments of passively switching laser are common in textbooks [18, 19]. More complete analyses have been reported in the literature by many authors [13-17]. For instance, Erneux [16] has proposed an asymptotic approximation of the periodic solution which based on singular perturbation techniques for the CO_2 laser parameters with different absorbers; Degnen [15] has reported on using the Lagrange multiplier technique to optimize the key parameters of Q-switching. The excited states absorption leads to additional loss in SA based lasers and can be an important consideration in their optimizations. Since SA does not bleach completely with this extra channel energy loss, therefore ignoring this effect in the theoretical treatment can lead to incorrect estimation of Q-switching laser parameters.

In this paper, a simple simulation model is presented based on solving numerically the passive Q-switching rate equations which has been modified to include the influence of the excited states SA photon absorption. The model is applied to analyze and optimize the performance of passively Q-switching $Nd:GdVO_4/Cr^{4+}$:YAG laser. The effects of the SA concentration and the output coupler reflectivity on optimizing the Q-switching laser characteristics are examined.

2. Rate Equations:

Considering the photon absorption of the SA excited states as a photon loss process and neglecting their spontaneous emission contribution and also the spontaneous emission of the gain medium, the rate equations for the laser photon density, gain medium population inversion and SA inversion are modified here to the following three-coupled equations:

$$\frac{d\Phi}{dt} = \frac{\Phi}{t_R}\{2\sigma LN - [2L_S(\sigma_S n + \sum_x \sigma_{Sx} n_x) + \ln(\frac{1}{R}) + \Gamma]\} \quad (1)$$

$$\frac{dN}{dt} = PK - \gamma c\sigma N\Phi - \frac{N}{\tau} \quad (2)$$

$$\frac{dn}{dt} = -\frac{A}{A_S} c\sigma_S n\Phi + \sum_x \frac{n_x}{\tau_{Sx}} \tag{3}$$

where Φ is the photon density; N is the population inversion of the laser gain medium; n and n_x are the population of the ground state and the excited state "x" of the SA, respectively where $n+\sum n_x = n_0$ the total population of the SA atoms; σ is the laser emission cross section; σ_S and σ_{Sx} are the absorption cross-sections of the ground state and the excited state of the SA; t_R is the cavity round trip transit time; $1/\tau$ is the decay rate of the upper laser level; τ_{Sx} is the life time of the excited state "x"; L is the length of the laser gain medium; L_S is the length of the SA crystal; γ corresponds to the net reduction in the population inversion resulting from the stimulated emission of a single photon where $\gamma = 1$ for an ideal four-level laser system and $\gamma = 2$ for an ideal three-level laser system; R is the output coupler reflectivity; Γ is the remaining round-trip cavity dissipative loss; P is the absorbed pump power; K is the pumping rate constant [13]; A is the effective laser beam area and A_S is the effective area of the SA.

Since for effective Q-switching operation the duration of the giant laser pulse is much shorter than the pumping or the relaxation times of either the laser gain medium or the SA, therefore we can be safely neglected them during the development of the giant laser pulse, hence Eqs. (2) and (3) can be simplified to,

$$\frac{dN}{dt} = -c\sigma\gamma\Phi N \tag{4}$$

$$\frac{dn}{dt} = -\frac{A}{A_S} c\sigma_S \Phi n \tag{5}$$

Dividing (4) on (5) and integrating, we get;

$$n = n_0 \left(\frac{N}{N_i}\right)^{\sigma_S A / \gamma\sigma A_S} = n_0 \left(\frac{N}{N_i}\right)^{\alpha} \tag{6}$$

where the parameter α indicates how fast the SA is bleached (the higher the value of α the faster the bleaching of the SA) and N_i is the initial population inversion density at the onset of Q-switching which can be determined from the condition that the roundtrip gain (the first term in the right hand side of (1)) is exactly equal to the roundtrip losses (the square bracket in (1)) just before the onset of Q-switching (*n* ≈ n_0),

$$N_i \cong \frac{1}{2\sigma L}(2\sigma_S L_S n_0 + \ln(\frac{1}{R}) + \Gamma) = \frac{1}{2\sigma L}\left(\ln(\frac{1}{T_0^{\,2}}) + \ln(\frac{1}{R}) + \Gamma\right) \qquad (7)$$

Continuing pumping, the number of photons within the cavity increases rapidly and results in saturation of the saturable absorber (n ≈ 0) with a corresponding threshold value of the population inversion N_{th} approximated to,

$$N_{th} \cong \frac{1}{2\sigma L}\left(2L_S \sum_x \sigma_{Sx} n_x + \ln(\frac{1}{R}) + \Gamma\right) = \frac{1}{2\sigma L}\left(\delta \ln(\frac{1}{T_0^{\,2}}) + \ln(\frac{1}{R}) + \Gamma\right) \qquad (8)$$

where $\delta = \frac{\sum_x \sigma_{Sx} n_x}{\sigma_S n_0}$. Dividing (1) by (4) and substituting (6) and (8) into the result, we obtain

$$\frac{d\Phi}{dN} = \frac{L}{L_C \gamma}\left[1 - \frac{(1-\delta)\ln(1/T_0^2)}{2\sigma L N}\left(\frac{N}{N_i}\right)^{\alpha} - \frac{\ln(1/R) + \delta \ln(1/T_0^2) + \Gamma}{2\sigma L N}\right] \qquad (9)$$

This equation can be integrated to yield

$$\Phi(N) = \frac{L}{L_C \gamma}\left[N_i - N - N_{th}\ln(\frac{N_i}{N}) - \frac{1-\delta}{2\sigma L \alpha}\ln(\frac{1}{T_0^2})\left(1 - \left(\frac{N}{N_i}\right)^{\alpha}\right)\right] \qquad (10)$$

At $N = N_{th}$, the photon density reaches its maximum and an expression for Φ_{max} can be obtained:

$$\Phi_{max}(N) = \frac{L}{L_C \gamma}\left[\frac{1-\delta}{2\sigma L}\ln(\frac{1}{T_0^2})\left(1 - \frac{1}{\alpha}\left(1 - \left(\frac{N}{N_i}\right)^{\alpha}\right)\right) - N_{th}\ln(\frac{N_i}{N_{th}})\right] \qquad (11)$$

Notice that δ must be less than unity otherwise a giant pulse will never develop and as δ approaches unity the Q-switching efficiency decreased. Also, since (9) at $N=N_i$ is equal to zero, therefore the criterion for a giant pulse to occur - $\left(\text{obtained from}: \left.\frac{d^2\Phi}{dN^2}\right|_{N=N_i}\right)$ - is given by:

$$[\alpha(1-\delta)-1]\ln(\frac{1}{T_0{}^2})-\ln(\frac{1}{R})-\Gamma>0 \tag{12}$$

where: $$\delta<1-\frac{\ln(1/RT_0^2)+\Gamma}{\alpha\ln(1/T_0^2)} \tag{13}$$

After releasing the giant Q-switched pulse the photon number in the cavity becomes very low ($\Phi\approx 0$) corresponding to a low inversion value N_f , at this point Eqn.(10) can be rewritten as:

$$N_i-N_f\cong N_{th}\ln(\frac{N_i}{N_f})+\frac{N_i}{\alpha}\left(1-\frac{N_{th}}{N_i}\right)\left(1-\left(\frac{N_f}{N_i}\right)^{\alpha}\right) \tag{14}$$

This expression is transcendental and can be solved numerically. When Ni and Nf are known, the output energy E_{out}, the average power P_{av} and the peak power P_{peak} of the Q-switched laser pulse can be calculated.

3. Numerical simulation:

Equations (1-3) have been solved numerically using Runge-Kutta method and a simple computer model is developed and applied to investigate the passive Q-switching performance of Nd:$GdVO_4$ laser with Cr^{4+}:YAG as a SA. The energy levels diagram of the Cr^{4+}:YAG crystal is shown in Fig.1 [1-4]. Noting that the transition between the ground state and the first excited state $^3T_2$ is solely responsible for the optical bleaching of the crystal [4]. The total population of SA atoms is $n_0=n+\sum n_x=n+n_T+n_E+n_{T1}+n_{A1}$. Because of the fast decay from levels T_1 and A_1 (τ_{T1} and τ_{A1} < 10ps [4]), it is assumed that the total population is only $n_0= n+n_T+n_E$. The experimental parameters used in this simulation are as follows [11]: the pump wavelength = 808 nm; the laser wavelength = 1064 nm; the laser cavity length = 60 mm; the laser rod dimensions = 3.5x3.5x4 mm^3; σ = 7.6x10^{-19} cm^2; σ(absorption) = 4.9x10^{-19} cm^2; τ = 90 µs; the SA crystal dimensions = 5x5x1.74 mm^3; σ_S = 3x10^{-18} cm^2; $\sigma_{ST}=\sigma_{SE}$ = 3x10^{-19} cm^2 [4];

$\tau_{ST} = 3x10^{-6}$ s; $\gamma = 1$; $\Gamma = 0.02$, the pump power P = 10.5W and effective area $A = A_S$.

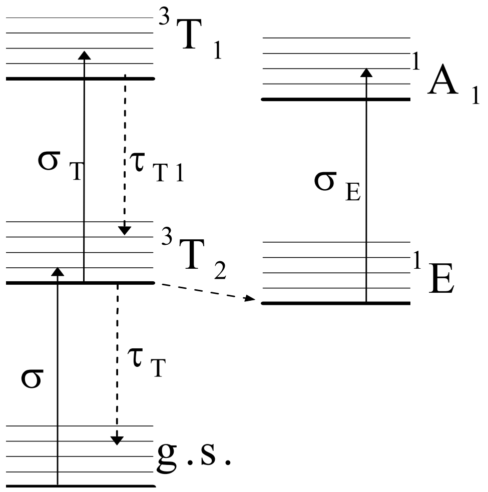
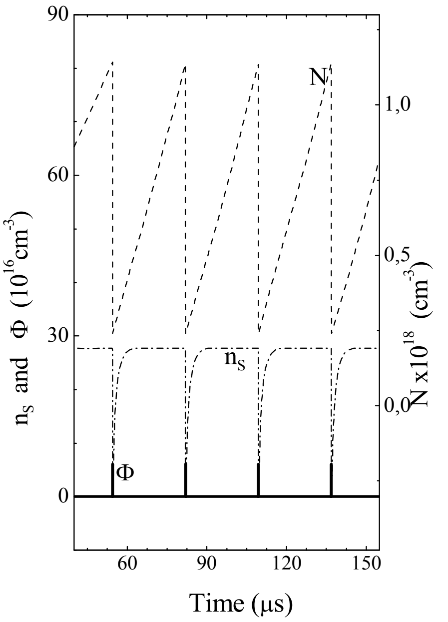


Fig. (1): Energy levels diagram of Cr4+:YAG.

The numerical results of the time dependencies of the laser population inversion *N,* the SA ground state population *n* and the photons density Φ are presented in Figure 2 (a, b), for initial transmission of Cr^{4+}:YAG T_0 = 75%, output coupler reflectivity R = 75%. Figure 2a presents a general view of these time dependent laser parameters, where the first giant pulse is developed at nearly 54µs after the pumping starts and the photon density peaks are developed with a time spacing of ~29µs corresponding to pulse repetition frequency PRF ≈ 30 kHz. Figure 2b focuses on the details of the temporal variations of these parameters near a peak of the photon density. In this figure, the photon density reaches a maximum value of $5.7x10^{15}$ cm^{-3} with a pulse width of ~15ns (FWHM) and pulse energy of ~80 µJ. Note here that the values of N_i , N_{th} and Φ_{max} estimated using the analytical Eqs. (7-10) are $1.2x10^{18}$, $4.9x10^{17}$ and $6x10^{16}$, respectively. These values are in reasonable agreement with those observed in Fig. (2b). The simulation results thus justify the derivation and the usefulness of these equations.



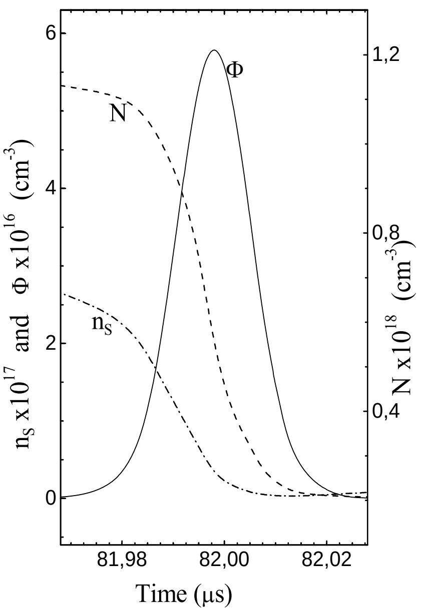


Fig. (2): Numerical simulation of passively Q-switching rate equations.
(a) General view
(b) details of temporal profiles for Nd:$GdVO_4$ laser population inversion "N", Cr^{4+}:YAG ground state population "n_S" and cavity photon density "Φ" at T_0 = 75% and R = 75%.

The effect of the initial transmission T_0 of Cr^{4+}:YAG on the Q-switch laser pulse energy, pulse width, average and peak powers are presented in Fig. 3 (a, b) at output coupler reflectivity R = 80%. It is obviously seen from these figures that as T_0 decreases the pulse energy and the peak power increase, whereas the pulse width and the average power decrease. This means that a narrower Q-switched pulse of a higher peak power and energy and lower average power is developed when a thicker or a higher doped concentration of the SA is used.

The calculated values of the pulse energy, pulse width, average and peak powers as functions of the output coupler reflectivity are presented in Fig. (4 a & b) for T_0 = 80% and compared to the corresponding experimental results of Liu et al. [11] (solid symbols). It is clear from these two figures that as the reflectivity of the output coupler decreases, the pulse energy and the peak power increase while the average power and the pulse width decreased in accordance with the experimental data of [11]. Reasonable agreement is observed between the calculated values and the corresponding experimental ones.

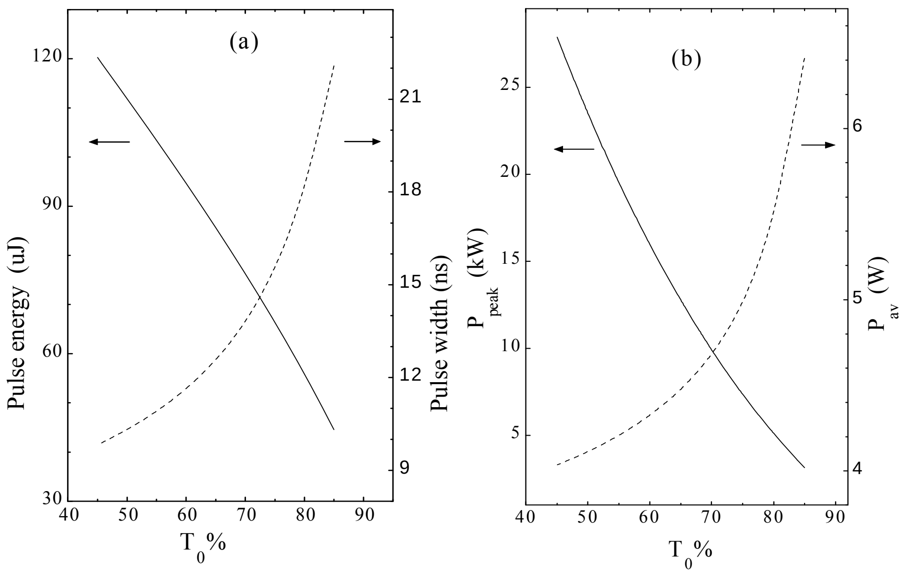


Fig. (3): Dependence of the Q-switched laser parameters: (a) output pulse energy and pulse width (FWHM) (b) Average and peak powers, on the initial transmission of Cr^{4+}:YAG at *R* = 80%.

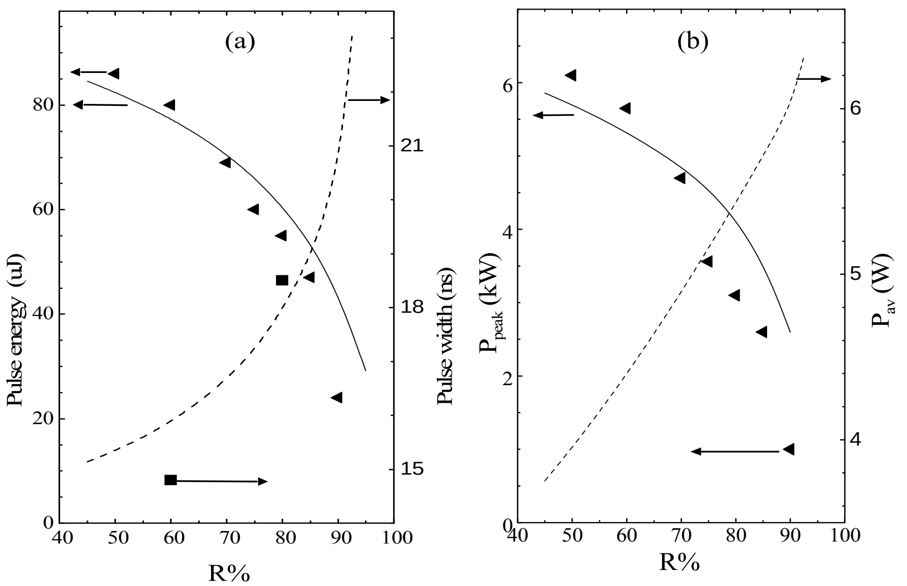


Fig. (4): Variation of (a) output pulse energy and pulse width (b) average and peak powers as functions of output coupler reflectivity at T_0 = 80%. The symbols represent the experimental results of [11].

Enhancement of the output pulse energy is one of the main targets in optimizing the performance of Q-switched laser systems. Therefore, in Fig. (5) the curves plot some defined pulse energies as a function of the output coupler reflectivity "R" and the small-signal SA transmission "T_0". The appropriate values of T_0 and R for various required output pulse energies can be immediately identified from this Figure.

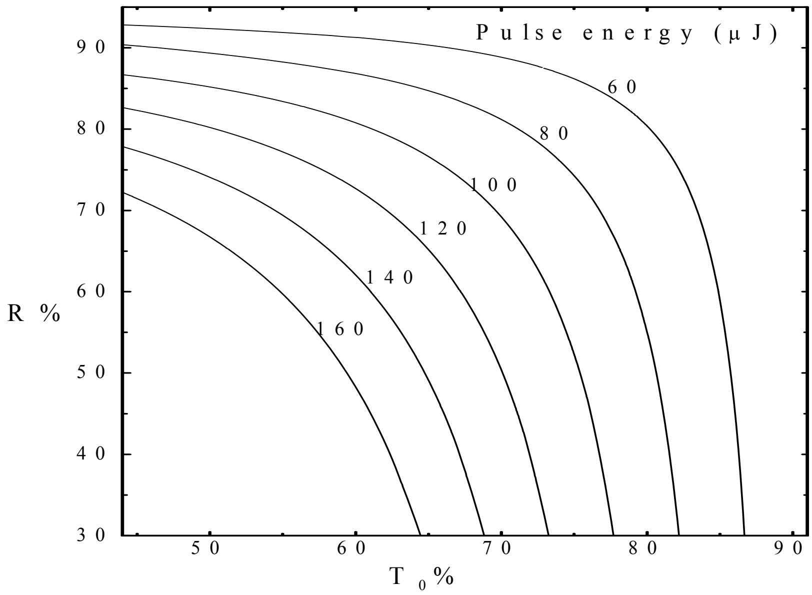


Fig. (5): Plots for some values of output pulse energies for various combinations of T_0 and R.

The dependence of the calculated pulse repetition frequency and pulse width on the pump power are presented in Fig. 6(a,b) for T_0 = 80 % and R = 60 %. The experimental results of Liu et al. [11] and Ng *et al*. [12] (solid and empty symbols, respectively) are also presented in the same figure for comparison. It is obvious that increasing the pump power yields shorter pulse widths with higher repetition frequency in accordance with the experimental results [10-12]. These calculations are repeated for two different values of T_0 = 80%, R = 80% and T_0 = 60%, R = 60% and the results are presented in Fig. (6 c) as a direct relationship between the pulse width and the pulse repetition frequency (at the same value of pump power). Fig. (6 c) shows clearly that shortening the pulse width is accompanied with increasing the repetition frequency, at defined values of T_0 and R. Also the lower values of output coupler reflectivity or SA initial transmissions exhibit shorter pulse width at the same repetition frequency and slower repetition frequency for the same pulse width.

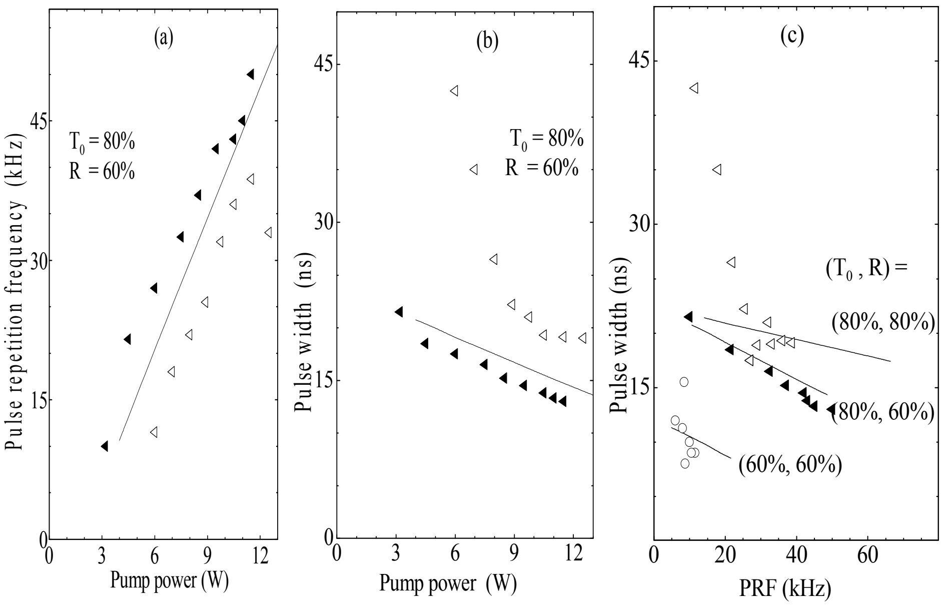


Fig. (6): Dependencies of: (a) the pulse width and (b) pulse repetition frequency on the pump power for T_0=80% and R=60%. The symbols are the experimental data of [11] (solid symbols) and [12] (empty symbols). (c) Variation of the pulse width against the pulse repetition frequency for: i) T_0=80%, R=80% ii) T_0=80%, R=60% and compared to the experimental ones of [11,12] iii) T_0=60% and R=60% and compared to [12].

4. Conclusion:

The passive Q-switching performance of Nd:$GdVO_4$ laser with Cr^{4+}:YAG solid state saturable absorber has been studied by solving numerically the passive Q-switching rate equations using Runge-Kutta method. The characteristics of the Q-switched laser parameters such as the pulse energy, the pulse width (FWHM), the average and peak powers and pulse repetition frequency have been explored as functions of the SA doping concentration and the reflectivity of the output coupler. Various combinations of the output coupler reflectivity and the small signal saturable absorber that can be used to enhance the output pulse energy are presented. The ranges of the obtained numerical results are in reasonable agreement with those obtained experimentally by other researcher [11, 12].

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